17301

15116

3 Hours / 100 Marks Seat No.

Instructions – (1) All Questions are *Compulsory.*

(2) Figures to the right indicate full marks.

(3) Assume suitable data, if necessary.

(4) Use of Non-programmable Electronic Pocket Calculator is permissible.

(5) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

(6) Use of Steam tables, logarithmic, Mollier's chart is permitted.

Marks

1. Attempt any FIVE of the following: **20**

a) At what point on the curve $y = e^x$, the slope is 1?

b) Find the radius of curvature of $y = e^x$ at $(0, 1)$.

c) Evaluate $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} \, dx$

d) Integrate w.r.t. x $\frac{\sin x}{\cos^2 x}$

e) Evaluate $\int x \, e^x \, dx$

f) Evaluate $\int \frac{1}{x(x+1)} \, dx$

Marks

g) Evaluate $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$

h) Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$ with X - axis.

i) Find order and degree of the following differential equation.

$$\frac{d^2y}{dx^2} = \sqrt{y + \left(\frac{dy}{dx}\right)^2}$$

j) Form the differential equation of the curve $y = ax^2$.

k) Three cards are drawn from well shuffled pack of cards. Find the probability that all of them are king.

l) Two coins are tossed simultaneously. Find the probability of getting at least one head.

2. Attempt any <u>FOUR</u> of the following: 16

a) Find the equation of tangent and normal to the curve

$2x^2 - xy + 3y^2 = 18$ at $(3, 1)$

b) Show that the radius of curvature at any point on the curve $y = a \log(\sec x/a)$ where a is constant is $a \sec(x/a)$.

c) Find the maximum and minimum value of $x^3 - 9x^2 + 24x$.

d) Evaluate $\int \cos^{-1}x \; dx$

e) Evaluate $\int \frac{(\tan^{-1}x)^3}{1+x^2}\,dx$

f) Evaluate $\int \frac{e^x}{(e^x - 1)(e^x + 1)}\,dx$

Marks

3. **Attempt any <u>FOUR</u> of the following:** **16**

a) Evaluate $\int_0^{\pi/2} \frac{\cos x}{4 - \sin^2 x} \, dx$

b) Evaluate $\int_0^{\pi/4} \log(1 + \tan x) \, dx$

c) Find the area of an ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ by integration.

d) Solve $\frac{dy}{dx} = \cos(x + y)$

e) Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$

f) Solve $(x + 1)\frac{dy}{dx} - y = e^x (x + 1)^2$

4. **Attempt any <u>FOUR</u> of the following:** **16**

a) Evaluate $\int_1^{5} \frac{\sqrt{9 - x}}{\sqrt{9 - x} + \sqrt{x + 3}} \, dx$

b) Evaluate $\int_0^{\pi/2} \frac{dx}{4 + 5\cos x}$

c) Find the area between the parabola $y^2 = 4x$ and the line $y = 2x + 3$

Marks

d) Solve $\frac{dy}{dx} = e^{2x+y} + x^2 e^y$

e) Solve $(2x + 3\cos y)\, dx + (2y - 3x \sin y)\, dy = 0$

f) Show that $y = A \sin mx + B \cos mx$ is a solution of differential equation $\frac{d^2 y}{dx^2} + m^2 y = 0$

5. Attempt any FOUR of the following: 16

a) A problem is given to the three students X, Y, Z whose chances of solving are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. Find the probability that:

(i) The problem is solved by each of them.

(ii) The problem is not solved by any of them.

b) If 30% of the bulbs produced are defective, find the probability that out of 4 bulbs selected:

(i) One is defective

(ii) At the most two are defective.

c) Using Poisson distribution, find the probability that the ace of spade will be drawn from a pack of well shuffled cards at least once in 104 consecutive trials.

d) Evaluate $\int \frac{dx}{2 + 3\cos x}$

e) $\int_0^1 x \tan^{-1} x \; dx$

f) Solve $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$

Marks

6. Attempt any <u>FOUR</u> of the following: 16

a) A bag contains 20 tickets numbered from 1 to 20. One ticket is drawn at random. Find the probability that it is numbered with multiple of 3 or 5.

b) A firm produces articles of which 0.1 % are defective, out of 500 articles. If wholesaler purchases 100 such cases, how many can be expected to have one defective? Given $e^{-0.5} = 0.6065$

c) I. Q.'s are normally distributed with mean 100 and standard deviation 15. Find the probability that a randomly selected person has:

(i) An I.Q. more than 130

(ii) An I.Q. between 85 and 115.

[z = 2, Area = 0.4772, z = 1, Area = 0.3413]

d) Divide 80 into two parts such that their product is maximum.

e) The equation of the tangent at the point (2, 3) on the curve $y = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.

f) Find the area of circle $x^2 + y^2 = 16$ by integration.